Sindh Univ. Res. Jour. (Sci. Ser.)

SINDH UNIVERSITY RESEARCH JOURNAL (SCIENCE SERIES)

Thermoelastic Waves Propagation in Rhombic Singony of the Classes mm2 and 222

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Abstract: The study of laws of wave propagation in elastic mediums due to thermomechanical effects is important in connection with geophysics, seismology, mechanics of composites, etc. Bound motion equations and heat conduction equations differ by difficulty and abundance of physical-mechanical parameters. Because of this reason theories regarding thermoelasticity are intensively being developed. In this paper a theoretical study has been carried out to investigate the propagation of thermoelastic waves in anisotropic mediums of rhombic syngony for the case of the second order axis symmetry and heterogeneity along Z axis. In the article, by means of analytical matrizant method, set of equations of motion for thermoelastic medium have been reduced to equivalent set of the first order differential equations.

Keywords: Anisotropic medium, thermoelasticity, Fourier heat equation, harmonic waves, matriciant.

1. INTRODUCTION

The theory of thermoelasticity deals with the study of dynamical interaction between thermal and mechanical fields in solid bodies and is of much importance in various engineering fields such as earthquake engineering, soil dynamics, aeronautics, nuclear reactors, ets. It is well known that the classical theory of thermoelasticity rests upon the hypothesis of the Fourier law of heat conduction, in which the temperature distribution is governed by a parabolic-type partial differential equation, *K.. Verma (2011)*. The theory predicts that a thermal signal is felt instantaneously everywhere in a body. This is unrealistic from the physical point of view, especially for short-time responses. To account for the effect of thermal relaxation, generalized thermoelasticity has been formulated on the basis of a modified Fourier law such that the temperature distribution is governed by a hyperbolic-type equation. Accordingly, heat transport in solids is regarded as a wave phenomenon rather than a diffusion phenomenon.

Recently the wave propagation in anisotropic inhomogeneous medium has been investigated by the new method of matricant. The method of matricant allows to investigate wave propagation in anisotropic medium with various physical and mechanical properties Nowacki,, *et a.l,(1957),S. Tleukenov (2004).*

The structure of matricant for the equation of motion in elastic medium and in thermo-mechanical medium has been established. Wave propagation in infinite and finite periodical inhomogeneous medium are studies.

In the paper *Tleukenov S (2004)*, waves propagating along an arbitrary direction in a heat conducting orthotropic thermoelastic plate are presented by utilizing the normal mode expansion method in generalized theory of thermoelasticity with one thermal relaxation time. In the paper Nowacki,, *et a.l,(1957)*, authors studied the interaction of free harmonic waves with multilayered medium in generalized thermoelasticity by utilizing the combination of the linear transformation formation and transfer matrix method approach. Solutions obtained are generaland and pertain to several special cases. For instance, dispersion characteristics for a multilayered.

2. THE MATRIX FORMULATION OF THE PROPAGATION OF THERMOELASTIC WAVES

Propagation of thermoelastic waves in anisotropic medium described by the equations of motion have to be solved together with the Fourier heat equation and the equation of heat flow, which have the form:

$$\frac{\partial\sigma_{XX}}{\partial X}+\frac{\partial\sigma_{XY}}{\partial Y}+\frac{\partial\sigma_{XZ}}{\partial Z}=\rho\frac{\partial^2 U_X}{\partial t^2}$$

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(1)

$$\frac{\partial\sigma_{XZ}}{\partial X}+\frac{\partial\sigma_{YZ}}{\partial Y}+\frac{\partial\sigma_{ZZ}}{\partial Z}=\rho\frac{\partial^2 U_Z}{\partial t^2}$$

$$\lambda_{ij}\frac{\partial\theta}{\partial x_j}=-q_i$$

(2)

$$\frac{\partial q_i}{\partial x_i}=-i\omega\beta_{ij}\varepsilon_{ij}-i\omega\frac{c_\varepsilon}{T_0}\theta$$

(3)

where σ_{ij} - stress tensor, ρ - density of the medium, λ_{ij} - thermal conductivity tensor, q_i - the vector of heat, ω - the angular frequency, β_{ij} - thermomechanical constants, $\beta_{ij}=\beta_{ji}$, ε_{ij} - the strain tens c_ε - specific heat or, at constant strain, $\theta=T\text{-}T_0$ - temperature increase compared with the temperature of the natural state T_0, $\left|\frac{\theta}{T_0}\right|\prec\prec 1$ for small deformations.

Physical and mechanical quantities are related by relation of Duhamel-Neumann:

$$\sigma_{ij}=c_{ijkl}\varepsilon_{kl}-\beta_{ij}\theta \tag{4}$$

Here c_{ij} is the elastic parameters, $c_{ijkl}=c_{jikl}=c_{ijlk}=c_{klij}$; ε_{kl} - the Cauchy tensor for small deformations.

For a rhombic class of crystals the ratio of

Duhamel - Neumann looks like:

$$\begin{pmatrix}\sigma_{xx}\\ \sigma_{yy}\\ \sigma_{zz}\\ \sigma_{yz}\\ \sigma_{xz}\\ \sigma_{xy}\end{pmatrix}=\begin{pmatrix}c_{11}&c_{12}&c_{13}&0&0&0\\ c_{12}&c_{22}&c_{23}&0&0&0\\ c_{13}&c_{23}&c_{33}&0&0&0\\ 0&0&0&c_{44}&0&0\\ 0&0&0&0&c_{55}&0\\ 0&0&0&0&0&c_{66}\end{pmatrix}\begin{pmatrix}\varepsilon_1\\ \varepsilon_2\\ \varepsilon_3\\ \varepsilon_4\\ \varepsilon_5\\ \varepsilon_6\end{pmatrix}-\begin{pmatrix}\beta_{11}&0&0\\ 0&\beta_{22}&0\\ 0&0&\beta_{33}\\ 0&0&0\\ 0&0&0\\ 0&0&0\end{pmatrix}\theta \tag{5}$$

Equations (1), (2), (3), (4) and (5) provide the relationship between the mechanical stress and the temperature as a function of the independent variables i-e the thermal field and deformation.

Thus, the relation (1) - (4) constitute a closed system of thermoelasticity equations, which describes the propagation of thermoelastic waves.

Based on the method of separation of variables in the case of a harmonic function of time:

$$\Big[U_i(x,y,z,t);\ \ \sigma_{ij}(x,y,z,t);\theta;q_z\Big]=$$

$$\Big[U_i(z),\sigma_{ij}(z),\theta;q_z\Big]e^{i(\omega t-mx-ny)}$$

(6)

The system of equations (1) - (4) reduces to a system of differential equations of first order with variable coefficients which describes the propagation of harmonic waves:

$$\frac{d\vec{W}}{dz}=B\vec{W} \tag{7}$$

Here $B=B\big[c_{ijkl}(z),\beta_{ij}(z),\omega,m,n\big]$ coefficient matrix whose elements contain the parameters of the medium in which waves propagate thermoelastic; m, n- components of the wave vector $\tilde{\kappa}$.

The vector $\vec{W}$ has the form:

$$\vec{W}(x,y,z,t)=\big[u_z(z),\sigma_{zz},u_x(z),\sigma_{xz},u_y(z),\sigma_{yz},\theta,q_z\big]^t\exp(i\omega t-imx-iny) \tag{8}$$

The symbol t indicates the transpose of the vector - a vector of strings - Column.

The system of differential equations for non-isotropic medium of a rhombic singony looks like:

$$\frac{dU_z}{dZ}=\frac{1}{c_{33}}\sigma_{zz}+\frac{c_{13}}{c_{33}}imU_x+\frac{c_{23}}{c_{33}}inU_y+\beta_{33}\theta$$

$$\frac{d\sigma_{zz}}{dZ}=-\rho\omega^2U_z+im\sigma_{xz}+in\sigma_{yz}$$

$$\frac{dU_x}{dZ} = \frac{1}{c_{55}} \sigma_{zx} + imU_z$$

$$\frac{d\sigma_{xz}}{dZ} = i m \frac{c_{13}}{c_{33}} \sigma_{zz} + \left[-\rho\omega^2 + m^2 \left(c_{11} - \frac{c_{13}^2}{c_{33}} \right) + c_{66} n^2 \right] U_x + m\, n \left(c_{12} + c_{66} - \frac{c_{13} c_{23}}{c_{33}} \right) U_y$$
$$+ \left(\frac{c_{13}}{c_{33}} \beta_{33} - \beta_{11} \right) im\, \theta$$

(9)

$$\frac{dU_y}{dZ} = \frac{1}{c_{44}} \sigma_{yz} + inU_z$$

$$\frac{d\sigma_{yz}}{dZ} = i n \frac{c_{23}}{c_{33}} \sigma_{zz} + mn \left[c_{12} + c_{66} - \frac{c_{13} c_{23}}{c_{33}} \right] U_x + \left(-\rho\omega^2 + m^2 c_{66} - \left(c_{22} - \frac{c_{23}^2}{c_{33}} \right) n^2 \right) U_y +$$
$$+ \left(\frac{c_{23}}{c_{33}} \beta_{33} - \beta_{22} \right) i n \theta$$

$$\frac{d\theta}{dZ} = -\frac{1}{\lambda_{33}} q_z$$

$$\frac{dq_z}{dZ} = -i\omega \frac{\beta_{33}}{c_{33}} \sigma_{zz} + \omega\, m \left(\frac{c_{13}}{c_{33}} \beta_{33} - \beta_{11} \right) U_x + \omega\, n \left(\frac{c_{23}}{c_{33}} \beta_{11} - \beta_{22} \right) U_y - i\omega \left(c_\varepsilon + \frac{\beta_{33}^2}{c_{11}} \right) \theta$$

The heterogeneity of the medium is assumed along Z. In constructing the coefficient matrix, *B* is used as a representation of the solution [5], the system of equations (1) - (4) are in the derivatives along the coordinate Z and the excluded components of the stress tensor have not been included in the boundary conditions. The multiplier $\exp(i\omega t - imx - iny)$ is omitted throughout.

The structure of the matrix and vector - column boundary conditions in the bulk case for the rhombic crystal system as well as the symmetry axis of the second order and heterogeneity along the Z axis are given by:

$$B = \begin{bmatrix} 0 & b_{12} & b_{13} & 0 & b_{15} & 0 & b_{17} & 0 \\ b_{21} & 0 & 0 & b_{24} & 0 & b_{26} & 0 & 0 \\ b_{24} & 0 & 0 & b_{34} & 0 & 0 & 0 & 0 \\ 0 & b_{13} & b_{43} & 0 & b_{45} & 0 & b_{47} & 0 \\ b_{26} & 0 & 0 & 0 & 0 & b_{56} & 0 & 0 \\ 0 & b_{15} & b_{45} & 0 & b_{65} & 0 & b_{67} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & b_{78} \\ 0 & -i\omega b_{17} & -i\omega b_{47} & 0 & -i\omega b_{67} & 0 & b_{87} & 0 \end{bmatrix};$$

$$\vec{W} = \begin{pmatrix} u_z \\ \sigma_{zz} \\ u_x \\ \sigma_{xz} \\ u_y \\ \sigma_{yz} \\ \theta \\ q_z \end{pmatrix} \qquad (10)$$

From the structure of the coefficient matrix (10) it evident that in the spatial case, the elastic waves of different polarization and the heat wave are interrelated.

The b_{ij} elements of the coefficient matrix *B* for a rhombic singony of the classes mm2 and 222 in a volume case look like:

$$b_{12} = \frac{1}{c_{33}}; b_{13} = \frac{c_{13}}{c_{33}} im; b_{15} = \frac{c_{23}}{c_{33}} in; b_{17} = \frac{\beta_{33}}{c_{33}};$$

$$b_{21} = -\omega^2 \rho;\ b_{24} = im;\ b_{26} = in$$

$$b_{34} = \frac{1}{c_{55}}; b_{43} = \left(c_{11} - \frac{c_{13}^2}{c_{33}} \right) m^2 + c_{66} n^2 - \omega^2 \rho; b_{45} = \left(c_{66} + c_{12} - \frac{c_{13} c_{23}}{c_{33}} \right) mn;$$

$$b_{47} = \left(\frac{c_{13}}{c_{33}} \beta_{33} - \beta_{11} \right) im$$

$$b_{56} = \frac{1}{c_{44}}; b_{65} = \left(c_{66} - \frac{c_{23}^2}{c_{33}} \right) n^2 + c_{66} m^2 - \omega^2 \rho; b_{67} = \left(\frac{c_{23}}{c_{33}} \beta_{33} - \beta_{22} \right) in;$$

$$b_{87} = -i\omega \left(\frac{\beta_{33}^2}{c_{33}} + \frac{c_\varepsilon}{T_0} \right); b_{78} = -\frac{1}{\lambda_{33}}.$$

The nonzero elements of the matrix of coefficients *B* b_{13}, b_{24} determine the mutual transformation of longitudinal and transverse X - polarized waves. Elements of b_{15} and b_{26} describe the relationship of transverse Y-polarization with the longitudinal wave. Nonzero element b_{45} defines the mutual transformation between the waves of transverse polarization.

The fact that the coefficient b_{17}:

$$b_{17} = \frac{\beta_{33}}{c_{33}}$$

means that the longitudinal wave is propagated from the thermoelastic effect. Where non-zero elements b_{47} and b_{67}:

$$b_{47} = \left(\frac{c_{13}}{c_{33}} \beta_{33} - \beta_{11} \right) im;$$

$$b_{67} = \left(\frac{c_{23}}{c_{33}} \beta_{33} - \beta_{22} \right) in$$

indicate the effect on the elastic wave transverse polarizations thermoelastic effect. At the same time describes the effect b_{47} thermoelastic effect on the elastic shear wave of the X-polarization, and b_{67} effects thermoelastic effect on the transverse wave Y-polarization.

Similarly, for the thermo-elastic waves propagating in an anisotropic medium of cubic symmetry the coefficient matrix is constructed in the bulk case and the analysis of matrix coefficients. We also obtain the structure of the matrix of coefficients in the propagation of thermoelastic waves in anisotropic medium of rhombic crystal systems in the plane XZ and YZ, defines the types of waves and the mutual transformation of waves of different polarizations.

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4. CONCLUSION

Differential equations system of the first order with variable coefficients that are made by means of variable separation method are made (solution is presented as a plane harmonic wave). Coefficients matrix for all seven types of anisotropic mediums of a rhombic singony of the classes mm2 and 222 for one, two and three dimensional cases are obtained.

ACKNOWLEDGEMENT

This is extended version of paper published as conference proceedings of ICCET 2013 held on March 5-7, 2013.

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